# Is the equity value of time really fair?

Guoqing Zhang and James Laird, Institute for Transport Studies, University of Leeds, Leeds LS2 9JT, UK

## 1 Introduction

A key element of benefit from transport infrastructure improvements is travel time savings. The value travellers attribute to travel time savings is dependent on a number of variables – but principally income and distance (Mackie et al., 2003; Abrantes and Wardman, 2011). However it is common practice internationally to use average values of time in appraisal (varying by journey purpose, vehicle type, day of week or time of day) rather than values which vary with the characteristics of the traveller (Odgaard et al, 2005). The UK and Scotland are no exception. In the UK and Scotland appraisal practice distinguishes between work and non-work trips (commuting and other non-work) and for non-work trips to use a standard value of travel time savings (VTTS) for all trips. These standard or equity values of VTTS are used in the Cost Benefit Analysis (CBA) as they are regarded as fair – preventing investment concentrating in high income areas. An alternative to the use of equity values of non-work time is to use local values - varying with income and trip distance - and weighting the results such that the benefits received by 'poor' people are weighted higher than those received by 'rich' people. This more sophisticated approach is consistent with economic principles and also the Treasury's Green Book, but does not form either part of STAG or its English counterpart webTAG. This more sophisticated approach is also a more onerous approach both analytically and from a data perspective. This is because it introduces additional calculations into the CBA, and also places a requirement on the analyst to know or estimate what the incomes of non-work travellers are (i.e. whether they are poor or rich).

In this paper we examine whether the use of the equity values of travel time savings is really fair – as it is argued they are. We firstly consider whether the use of local VTTS – values reflecting the characteristics of the traveller and their trip – give different results from CBA results calculated using an equity value. We also compare the CBA results with an equity value with results calculated using income weights derived using the Treasury's Green Book method. Finally as the equity VTTS implies a weighting between low and high income travellers we estimate what this is in different scenarios and examine whether there is any consistency in the weights and whether the weights appear excessive. It is argued for example that weights for poor people that exceed twice the weight for rich people are inappropriate from a policy perspective as there exist better policy instruments for ensuring an equitable distribution of wealth, such as welfare benefit system (Boardman et al., 2006). To do this we use firstly a synthetic analysis, and secondly data from two Scottish inter-urban road schemes. To derive an income profile for travellers we use Monte Carlo simulation fitted to observed income profiles for different areas.

This paper is structured as follows: Chapter 2 briefly summarises the methodology; Chapter 3 presents the synthetic analysis, which shows the economic impact of using local/regional values of time and the method of deriving implied weights; Chapter 4 presents the Scottish inter-urban case studies, which estimate regional user benefits and clarifies the appropriateness of the implied weights; whilst Chapter 5 sets out our conclusions.

## 2 Methodology

We adopt a two stage method. In the first stage we use a set of synthetic or hypothetical schemes to understand the issues associated with a distributional analysis for non-work time. In the second stage we apply the same method but to two inter-urban Scottish case studies.

### 2.1 Stage 1: Synthetic analysis

For the synthetic analysis we consider we simulate a set of 10,000 trips making firstly a 10 mile journey and experiencing a 10 minute time saving (Scenario 1) and then making a journey that was dependent in length on household income, but also receiving a 10 minute time saving (Scenario 2). This is undertaken for seven different locations - i.e. with different income profiles. The seven locations are: an average UK location, an average Scottish location and an average location in each of the following 5 English regions: East Midlands, Yorkshire & Humberside, London, West Midlands, and South West, The steps adopted in the analysis are set out below.

**Deriving household income.** For each of the hypothetical schemes Monte Carlo simulation was used to derive a set of household incomes. Data from the Family Resources Survey (FRS) 2011/12 (Anon, 2013c Table 2.5), which is one of the main sources of UK income statistics (Francesconi et al., 2009) is used to give the proportion of households in each of the income quintiles. A range of distributions were then fitted using MATLAB 2012. The distribution that best fitted the observed income profile was selected (based on maximum log likelihood). For each income quintile midpoints were used in the distribution fitting. For the upper income quintile a value of £78,300 was used as representing the average income for the richest households of the top quintile (Anon, 2013b). For the lowest income quintile a minimum household income of £8,000 was assumed. The fitting process is illustrated in Figure 1 for Scotland, with the results being detailed in Appendix A. The best fitting distributions were found to be gamma distributions, aside from the Birnbaum-Saunders distribution for East Midlands.

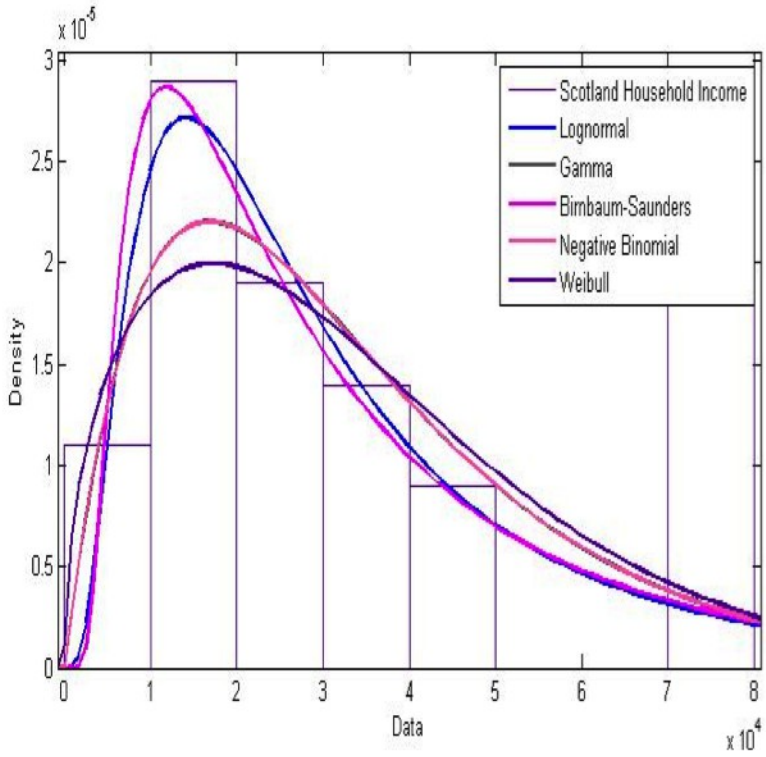


**Figure 1: Distribution fitting for Scotland**

**Deriving trip length.** In scenario 1 of the synthetic analysis all trips are assumed to have the same trip length – 10 miles. In scenario 2 the trip length is taken to vary with household income. This is because travelling is expensive and poor people are inclined to travel less compared to rich people even though their families' car ownership is the same. Following Anon (2012b Figures 3.6a-b) distance per trip by car driver and car passenger by household real income quintile can be set out as in Table 1. A glance at the table informs that average trip distances of both car driver and passenger diminish as income decreases. Combining these values using average vehicle occupancy[1] gives Table 2.

**Table 1 Distance per trip by household real income quintile (miles)**

| | Lowest real income level | Second level | Third level | Fourth level | Highest real income level |
|---|---|---|---|---|---|
| Car Driver | 6.4 | 6.8 | 7.5 | 8.5 | 11.0 |
| Car Passenger | 6.9 | 8.1 | 8.2 | 9.1 | 10.5 |

**Table 2 Updated distance per trip by household real income quintile**

| | Lowest real income level | Second level | Third level | Fourth level | Highest real income level |
|---|---|---|---|---|---|
| Car | 6.6 | 7.3 | 7.8 | 8.7 | 10.8 |

**Deriving VTTS values per trip.** The next step was to assign a VTTS for each trip based on household income and trip length. To do this VTTS values in Mackie et al. (2003 Table 22 and 23), were updated to 2010 prices, and combined into a single non-work VTTS table.

The updating process used the relationship derived by Mackie et al (2003) and described in TAG Unit 3.10.2C (Anon, 2013d) for the derivation of local or regional VTTS. This is as shown in Equation 1 and gives the VTTS values in Tables 3 and 4 by income and trip distance. An important point to note is that only the cells highlighted in blue are those which have a value higher than the UK equity value of time (10.77 pence per minute for commuting and 9.52 pence per minute for other non-work) – as can be seen these are typically associated with high incomes or long trip distances. Combining these tables using average proportions[2] (Anon, 2012d, Table 8) gives an average non-work VTTS table by income and distance as set out in Table 5. The equity value of time here is 9.78 pence/minute[3], and the cells highlighted in green have VTTS values greater than the equity value, and those in white have values below the equity value.

$$VoT = G^{\alpha} K \left[\frac{\beta_{\tau}}{\beta_{c}}\right] \left(\frac{Inc'}{Inc'_{0}}\right)^{\eta_{Inc}} \left(\frac{D}{D_{0}}\right)^{\eta_{c}}$$

Equation 1

Where:

- $G^{\alpha}$: correction factor for growth of real GDP per capita between 1994 and 2010 with α standing for the elasticity of VoT with regard to income
- K: correction factor for inflation between 1994 and 2010

1 Table 4 in TAG Unit 3.5.6 (Anon, 2012d) tells that commuting and other car occupancy per trip are 1.14 and 1.79 in 2000 separately. Therefore, the commuting and other car passenger occupancy are 11.61% and 41.72% in 2010, which gives new car occupancy per trip of 1.13 and 1.72. Multiplying these two weights 0.2137 and 0.7863 produces a 1.59 for non-work car occupancy per trip.

2 20.3% for commuting and 74.7% for other non-work

3 Equity VoT = (6.46 (commuting) * 0.2137 + 5.71 (other) * 0.7863)/60 = 9.78 pence per minute

$\beta_T$: time coefficient, which is -0.10098 for commuting trip purpose and -0.082918 for other trip purpose according to Mackie et al. (2003)

$\beta_c$: cost (distance) coefficient, which is -0.024729 for commuting trip purpose and -0.022275 for other trip purpose (Mackie et al., 2003)

Inc': average household income in £'000 per year

$Inc'_0$: average household income

$\eta_{Inc}$: income elasticity

$\eta_c$: cost (distance) elasticity

D: average distance in miles

D0: the conversion of £1 cost to distance assuming a cost of 13.2p/mile

# Table 3: VoTs for commuting purpose in 2010 price

| Distance band | Average trip distance (miles) (source NTS 2008-2010) | £1,000 - £4,999 | £5,000 - £9,999 | £10,000 - £14,999 | £15,000 - £19,999 | £20,000 - £24,999 | £25,000 - £29,999 | £30,000 - £34,999 | £35,000 - £39,999 | £40,000 - £49,999 | £50,000 - £59,999 | £60,000 - £69,999 | £70,000 or more | £1,000 + |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Average income (£k) [Source: FRS 2009]** | | 3.02 | 8.20 | 12.54 | 17.46 | 22.37 | 27.28 | 32.41 | 37.29 | 44.68 | 54.45 | 64.32 | 92.47 | 26.57 |
| **Under 1 mile** | 0.5 | 0.9 | 1.3 | 1.5 | 1.7 | 1.9 | 2.0 | 2.1 | 2.2 | 2.4 | 2.5 | 2.7 | 3.1 | 2.0 |
| **1 to under 2 miles** | 1.3 | 1.3 | 1.8 | 2.2 | 2.4 | 2.7 | 2.8 | 3.0 | 3.2 | 3.4 | 3.6 | 3.9 | 4.4 | 2.8 |
| **2 to under 3 miles** | 2.2 | 1.6 | 2.3 | 2.7 | 3.1 | 3.4 | 3.6 | 3.8 | 4.0 | 4.3 | 4.6 | 4.9 | 5.6 | 3.6 |
| **3 to under 5 miles** | 3.6 | 2.0 | 2.9 | 3.4 | 3.8 | 4.1 | 4.4 | 4.7 | 5.0 | 5.3 | 5.7 | 6.0 | 6.9 | 4.4 |
| **5 to under 10 miles** | 6.8 | 2.6 | 3.8 | 4.4 | 4.9 | 5.4 | 5.8 | 6.2 | 6.5 | 6.9 | 7.4 | 7.9 | 9.0 | 5.7 |
| **10 to under 15 miles** | 11.7 | 3.3 | 4.7 | 5.5 | 6.2 | 6.8 | 7.3 | 7.8 | 8.2 | 8.7 | 9.3 | 9.9 | 11.3 | 7.2 |
| **15 to under 25 miles** | 18.3 | 4.0 | 5.7 | 6.6 | 7.5 | 8.2 | 8.8 | 9.3 | 9.8 | 10.5 | 11.3 | 12.0 | 13.6 | 8.7 |
| **25 to under 35 miles** | 28.7 | 4.8 | 6.9 | 8.0 | 9.1 | 9.9 | 10.6 | 11.3 | 11.9 | 12.7 | 13.6 | 14.5 | 16.5 | 10.5 |
| **35 to under 50 miles** | 40.1 | 5.6 | 8.0 | 9.3 | 10.4 | 11.4 | 12.2 | 13.0 | 13.7 | 14.6 | 15.7 | 16.7 | 19.0 | 12.1 |
| **50 miles +** | 72.3 | 7.1 | 10.2 | 11.9 | 13.4 | 14.6 | 15.7 | 16.7 | 17.5 | 18.7 | 20.1 | 21.3 | 24.3 | 15.5 |
| **All distances** | 9.8 | 3.1 | 4.4 | 5.1 | 5.8 | 6.3 | 6.8 | 7.2 | 7.6 | 8.1 | 8.7 | 9.2 | 10.5 | 6.7 |

# Table 4: VoTs for other non-work purpose in 2010 price

| Distance band | Average trip distance (miles) (source NTS 2008-2010) | £1,000 - £4,999 | £5,000 - £9,999 | £10,000 - £14,999 | £15,000 - £19,999 | £20,000 - £24,999 | £25,000 - £29,999 | £30,000 - £34,999 | £35,000 - £39,999 | £40,000 - £49,999 | £50,000 - £59,999 | £60,000 - £69,999 | £70,000 or more | £1,000 + |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Average income (£k) [Source: FRS 2009]** | | 3.02 | 8.20 | 12.54 | 17.46 | 22.37 | 27.28 | 32.41 | 37.29 | 44.68 | 54.45 | 64.32 | 92.47 | 26.57 |
| Under 1 mile | 0.5 | 1.9 | 2.2 | 2.4 | 2.5 | 2.6 | 2.7 | 2.7 | 2.8 | 2.9 | 3.0 | 3.1 | 3.2 | 2.7 |
| 1 to under 2 miles | 1.2 | 2.5 | 2.9 | 3.1 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 3.9 | 4.0 | 4.2 | 3.5 |
| 2 to under 3 miles | 2.2 | 3.0 | 3.5 | 3.7 | 3.9 | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.7 | 4.8 | 5.1 | 4.2 |
| 3 to under 5 miles | 3.5 | 3.5 | 4.0 | 4.3 | 4.6 | 4.7 | 4.9 | 5.0 | 5.1 | 5.3 | 5.4 | 5.6 | 5.9 | 4.9 |
| 5 to under 10 miles | 6.5 | 4.2 | 4.9 | 5.3 | 5.5 | 5.8 | 5.9 | 6.1 | 6.2 | 6.4 | 6.6 | 6.8 | 7.2 | 5.9 |
| 10 to under 15 miles | 11.6 | 5.0 | 5.9 | 6.3 | 6.6 | 6.9 | 7.1 | 7.3 | 7.5 | 7.7 | 7.9 | 8.1 | 8.6 | 7.1 |
| 15 to under 25 miles | 18.2 | 5.8 | 6.8 | 7.3 | 7.6 | 7.9 | 8.2 | 8.4 | 8.6 | 8.9 | 9.1 | 9.4 | 9.9 | 8.2 |
| 25 to under 35 miles | 28.5 | 6.7 | 7.8 | 8.3 | 8.8 | 9.1 | 9.4 | 9.7 | 9.9 | 10.2 | 10.5 | 10.8 | 11.4 | 9.4 |
| 35 to under 50 miles | 40.7 | 7.5 | 8.7 | 9.3 | 9.8 | 10.2 | 10.6 | 10.8 | 11.1 | 11.4 | 11.8 | 12.1 | 12.8 | 10.5 |
| 50 to under 100 mile | 68.3 | 8.8 | 10.3 | 11.0 | 11.6 | 12.0 | 12.4 | 12.8 | 13.0 | 13.4 | 13.8 | 14.2 | 15.0 | 12.4 |
| 100 to under 200 miles | 134.8 | 10.9 | 12.7 | 13.6 | 14.3 | 14.9 | 15.4 | 15.8 | 16.2 | 16.6 | 17.1 | 17.6 | 18.6 | 15.3 |
| 200 miles and over | 259.6 | 13.4 | 15.7 | 16.7 | 17.6 | 18.3 | 18.9 | 19.4 | 19.9 | 20.4 | 21.1 | 21.6 | 22.9 | 18.8 |
| All distances | 8.0 | 4.5 | 5.2 | 5.6 | 5.9 | 6.1 | 6.3 | 6.5 | 6.6 | 6.8 | 7.0 | 7.2 | 7.6 | 6.3 |

**Table 5 Non-work VTTS lookup table**

| Income band / Distance band | £1,000 - £4,999 | £5,000 - £9,999 | £10,000 - £14,999 | £15,000 - £19,999 | £20,000 - £24,999 | £25,000 - £29,999 | £30,000 - £34,999 | £35,000 - £39,999 | £40,000 - £49,999 | £50,000 - £59,999 | £60,000 - £69,999 | £70,000 or more |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Under 1 mile | 1.7 | 2.0 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 | 3.2 |
| 1 to under 2 miles | 2.2 | 2.7 | 2.9 | 3.1 | 3.2 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 4.0 | 4.3 |
| 2 to under 3 miles | 2.7 | 3.2 | 3.5 | 3.7 | 3.9 | 4.1 | 4.2 | 4.3 | 4.5 | 4.7 | 4.8 | 5.2 |
| 3 to under 5 miles | 3.1 | 3.8 | 4.1 | 4.4 | 4.6 | 4.8 | 5.0 | 5.1 | 5.3 | 5.5 | 5.7 | 6.1 |
| 5 to under 10 miles | 3.9 | 4.7 | 5.1 | 5.4 | 5.7 | 5.9 | 6.1 | 6.3 | 6.5 | 6.8 | 7.0 | 7.6 |
| 10 to under 15 miles | 4.7 | 5.6 | 6.1 | 6.5 | 6.9 | 7.1 | 7.4 | 7.6 | 7.9 | 8.2 | 8.5 | 9.2 |
| 15 to under 25 miles | 5.4 | 6.6 | 7.1 | 7.6 | 8.0 | 8.3 | 8.6 | 8.9 | 9.2 | 9.6 | 9.9 | 10.7 |
| 25 to under 35 miles | 6.3 | 7.6 | 8.3 | 8.9 | 9.3 | 9.7 | 10.0 | 10.3 | 10.7 | 11.2 | 11.6 | 12.5 |
| 35 to under 50 miles | 7.1 | 8.6 | 9.3 | 10.0 | 10.5 | 10.9 | 11.3 | 11.6 | 12.1 | 12.6 | 13.1 | 14.1 |
| 50 to under 100 mile | 8.4 | 10.3 | 11.2 | 12.0 | 12.6 | 13.1 | 13.6 | 14.0 | 14.6 | 15.2 | 15.7 | 17.0 |
| 100 to under 200 miles | 10.1 | 12.2 | 13.2 | 14.1 | 14.8 | 15.4 | 16.0 | 16.5 | 17.1 | 17.8 | 18.4 | 19.8 |
| 200 miles and over | 12.0 | 14.5 | 15.7 | 16.7 | 17.5 | 18.2 | 18.8 | 19.4 | 20.1 | 20.9 | 21.6 | 23.2 |

**Calculating the user benefit.** A fixed trip matrix is assumed. The time saving is for each trip is therefore multiplies by the VTTS for that trip to give the benefit per trip. This is summed over all trips to give the total benefit. This is done for local and equity VTTS values.

**Estimating distributional weights.** In this case, households in each region are divided into 2 groups around the median income. Distributional weights for the low income and high income groups are estimated following Green Book guidance (Anon, 2003). These weights are used to weight the benefits derived from using the local/regional VTTS – which gives us a distributionally corrected estimate of the economic benefit. By comparing the local benefits to the benefits derived from using the standard equity value we also can derive the implied 'distributional' weights implicit in the application of the equity VTTS.

### 2.2 Scottish case studies

These are 2 significant regional trunk road upgrade schemes with potential implications for the economic strength and vitality of cities in Scotland. Data provided for do-minimum and each scheme are: sample size (707* 707 TMFS zones), population in each zone, SIMD (Scottish Index of Multiple Deprivation) zone ranking, SIMD income domain rank and demand for car trips as well as their distance and in-vehicle journey times. The modelled (forecast) year is 2022. Each matrix represents AM peak hour (08.00-09.00). The steps that were undertaken were as follows.

**Deriving household income.** Household income is not available for each trip that is made. It is therefore synthesised by using zone level data as a proxy for the income of travellers from that zone. As is said in the TAG Unit (Anon, 2011e), *information on household income at a small area is through commercially available data*. As an alternative to attain relative income level or deprivation, IMD data is accessible and free of charge. Scotland specific IMD is SIMD, which is aforementioned. To be more specific, with 6505 wards, which almost have the same population, in Scotland, there are 7 domains included, which are *employment; income; health; education, skills & training; geographic access to services; crime and housing* (Anon, 2012a). When attention is paid to the 707 TMFS (Transport model for Scotland) zones, each zone has about 9 wards.

It is assumed that all households in a zone have the same income. According to Scotland – Household Taxes and Benefits: update (Anon, 2012c), 71 same household incomes, which are before tax and after benefit as is given in the table below, are allocated to households in first nine deciles while 68 same incomes are given to households in the last decile.

**Table 6 Average household income for each decile**

| Decile | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Household Income | 17033 | 22723 | 26039 | 29301 | 32884 | 40307 | 47198 | 53930 | 65788 | 113096 |

Once again MATLAB is used to fit a distribution to this observed profile. Appendix B contains the details.

Using Monte Carlo simulation almost 500,000 household incomes were simulated. These were then ranked and grouped together. Each group comprised 707 households. Each group was allocated to a zone based on the SIMD income domain ranking (from smallest to largest). The mean value for each group was then used as the household income for that zone.

**Deriving demand, trip distance, journey times.** These data are taken directly from the transport model data for the do-minimum and 2 do-something scenarios.

**Deriving VTTS values per trip.** The next step was to assign a VTTS for each trip based on household income for the zone (simulated) and trip length (from the data). This used Table 5 above.

**Calculating user benefits.** In contrast to the synthetic analysis demand varies between the Do Minimum and the Do Something. The rule of half is therefore employed to estimate user benefits for each origin-destination movement.

User Benefit = $CS_1 - CS_0 = ½ \sum (PC^0_{ij} – PC^1_{ij})*(T^0_{ij} + T^1_{ij})$,

where:

CS consumer surplus;

PC perceived cost from origin i to destination j, and in this case, PC= unit VoT*In vehicle journey time;

$T_{ij}$ number of trips from i to j;

0 Do-minimum scenario;

1 Do-something scenario.

**Deriving implied distributional weights.** Similar to synthetic analysis, the sample is divided into 2 income groups. Instead of outputting a weight for the disadvantaged, ratios of weight for the disadvantaged to weight for the advantaged are estimated after introducing a restriction that weight for the disadvantaged is bigger than weight for the advantaged. And comparison with the result calculated by the method in Green Book is made as well.

## 3 Synthetic Analysis

### 3.1 Scenario 1: Fixed trip distance of 10 miles with non-work journey purpose, and every household only produces one trip enjoying 10 minutes' reduction a hypothetical scheme brings

Firstly, the benefit of 10,000 households from one of the hypothetical schemes, say the Yorkshire & Humberside scheme, are modelled. Aggregating across all trips this gives a benefit of £7,190.28. This process is then applied to the schemes in the other regions. Then, after allocating the same amount of households, which is 10,000, to 4 other regions, Scotland and UK, time saving benefits can be calculated. Equity value, which is 9.78 pence/minute for non-work purpose in the UK practice now, gives a benefit of £9,780. The results are presented in Figure 2.

As the VTTS per trip is only larger than the equity value of time for trips that exceed 25 miles (see Table 5) we can see from Figure 2 that all of our benefits derived using local/regional values are lower than that derived from using the equity value of time. This is even the case for the scheme in London. In detail, the scheme in London enjoys the biggest benefit from using local/regional values of £7,453.97 as its users have the highest mean income of the different schemes considered. East Midlands in contrast has the lowest benefit of £7092.59 for the same amount of time savings since its users belong to the poorest of the different regions considered.

It is suggested by the Green Book (Anon, 2003), *each monetary cost and benefit should be weighted according to the relative prosperity of those receiving the benefit or bearing the cost*. We therefore estimate the weights that should be applied to different income groups. Using the scheme in London as an example, after sorting random household incomes, the first 5,000 of the 10,000 households are defined as the low-income group with the remaining 5,000 households treated as the high-income group. This gives distributional weights of 1.63 for the low-income group and 0.62 for high-income

group[4]. The relative weight of the low-income group is 2.63[5], which goes against Boardman's view that *distributional weights assigned to the disadvantaged should not exceed 1.5 or 2 in value if the value for the advantaged is set to 1* (Boardman, 2006), while weight is set to 1 for the high-income group. The time saving benefit using regional value of time for low-income and high-income groups are £3,321.66 and £4,132.30. By multiplying two weights above, by their corresponding benefits, the new benefit using local value of time but weighted is £7,976.34[6], which is still 18% lower than equity value of time benefit.

We can also estimate the implied ratio of weights associated with the equity value of time – which is 1.70. This is because by multiplying the low income benefits (£3,321.66) by 1.70 and adding them to the high income benefits (£4,132.30) we get the equity VTTTS benefit (£9,780.00). In this specific case, UK equity value of time is not giving sufficient weight to the low-income group, compared to the Green Book approach. Whilst doing this it does however appear to overestimate the benefit due to the fact that the equity VTTS implies quite long trip distances.

**Figure 2 Comparison of different benefit levels (£)**

### 3.2 Scenario 2: trip distance varying with non-work journey purpose and 10 minutes' saving per trip for each household a hypothetical scheme brings

In reality, people's travel characteristics are not what are described in Scenario 1. Allowing trip distances to vary with income we get results as depicted in Figure 3.

Each household is assumed to generate one trip, which is same with scenario 1, and 3 levels of benefits of 10,000 households in each region can be illustrated in the following figure.

**Figure 3 Comparison of different benefit levels (£)**

Even though London enjoys the highest benefit (both Level 1 and 2) in this case, they are still 30% and 32% lower than the equity VoT benefit. Ranks for each region are demonstrated in the table 7.

**Table 7 Rank of each region in Scenario 2**

| | Rank using benefit Level 1 | Rank using benefit Level 2 |
|---|---|---|
| East Midlands | 7 | 4 |
| Yorkshire & Humberside | 5 | 6 |
| London | 1 | 1 |
| West Midlands | 6 | 7 |
| South West | 3 | 3 |
| Scotland | 4 | 5 |

4 The median income of London, low-income group and high-income group are £32,182.39 per year, £19,726.80 per year and £51,966.02 per year. In line with the practice in the Green Book, the utility function of household income is expressed as U = log C while C represents income and U means utility. The distributional weight is the ratio of the marginal utility of a group to average marginal utility. As a consequence, the marginal utilities, which is U' = 1/C, of London, low-income group and high-income group are 3.11E-05, 5.07E-05 and 1.92E-05 respectively, which produces a distributional weight of 1.63 for low-income group and 0.62 for high-income group.

5 All the rest have a weight bigger than 2.5 for low-income group if high-income group has a weight of 1.

6 1.63*3321.66+0.62*4132.3=7976.34

| | | |
|---|---|---|
| UK | 2 | 2 |

The key finding here therefore is that regional values of time will give rise to much lower time saving benefits than applying the UK equity value of time when trip lengths are short. This has policy implication for appraisals of transport projects in urban areas.

## 4 Scottish Case Studies

### 4.1 Analysis of Scheme 1

#### 4.1.1 Data characteristics

The rankings provided are a proxy for income and are from 1 to 707 with an increase in number as the zone becomes richer. In the demand matrix, the minimum origin-destination demand is 0 car and the maximum one is 1478 cars and the maximum is trips of intra-zone ranked 579 by income domain while with an average of 366 cars, demands of each zone have a upper bound of 1760 cars in zone ranked 648 and a lower bound of 26 cars in zone ranked 420, as is shown in the figure below.

**Figure 4 AM peak zone demand of Scheme 1**

The floor and ceiling of in-vehicle time are 0 minute and 1651 minutes that is from zone ranked 496 to zone ranked 579. Distance matrix reveals that the minimum value is 0 mile while the longest one is 555.7 miles, which is from the zone ranked 352 to the zone ranked 579. As a result, average trip distance and time of Scheme 1, which are demand weighted, can be displayed in the figures below.

**Figure 5 Demand weighted average time of Scheme 1**

**Figure 6 Demand weighted average distance of Scheme 1**

In terms of demand weighted average in-vehicle time, it's clear that right half zones in high income domain ranking, which have a peak value of 98 minutes in zone ranked 496, travel longer per trip than those in low income domain. What's more, most average time concentrates between 10 minutes and 40 minutes. Likewise, with an average of 11.03 miles, demand weighted average trip distances in the second half are inclined to be farther than the first half. Besides, most trip distances are from 5 miles to 20 miles.

#### 4.1.2 Time savings

As far as time saved for each OD pair is concerned, this scheme will increase travellers' journey times by at most 14 minutes from zone ranked 29 to zone ranked 152 with a trip distance of 6.96 miles, while it can save motorists a maximum of 61 minutes form zone ranked 560 to zone ranked 204 with a trip distance of 29.11 mile. When it comes to time saved of each zone, it is demonstrated in the following chart. Here, time saved of each zone[7] is also calculated using rule of half.

7 Time saved of zone i = ½ $\sum (T^0_j - T^1_j)*(D^0_j + D^1_j)$ where $T_j$ is journey time from origin i to destination j; $D_j$ is demand from origin i to destination j; 0 is do-minimum scenario and 1 is do-something scenario.

**Figure 7 Time saved of each zone**

It's apparent that zones that belong to wealthier half will enjoy much larger time savings with a largest saving of 296 minutes for zone ranked 560 though zone ranked 664, which will suffer from the largest time loss of 39 minutes, also comes from the wealthier half. Hence, the maximum time loss at the zone level is not from the same zone that has a maximum time loss at the OD pair level, as is discussed in the last paragraph. In addition, the average time saved for each zone will be 15 minutes. User benefit of each OD pair is given by multiplying time saved in each cell by unit VoT, and sum them across the whole matrix to obtain the final regional user benefit of the scheme, which will be £1,298.07 for AM peak hour whereas equity value of time produces a user benefit of £1,045.45 for the same time period. The question arises with regard to whether household income or trip distance leads to the difference. Consequently, both income and trip distance effects are tested in the following part.

With demand weighted average distance of 10.21 miles, demand weighted average income of £45,190.84 per year or total distance weighted average income £46,775.01 per year, which is adopted to calculate the equity value of time in TAG Unit 3.5.6 (Anon, 2012d), can be applied to create a same Scotland specific 'equity value', which is 7.89 pence/minute, as they fall into the same income band. Total distance weighted average income is higher than demand weighted average income because motorists with higher incomes are prone to travelling further. The reason why Scotland specific equity value of time is smaller than the UK one is that Scotland is slightly poor in the UK context although demand weighted average distance is longer than Great Britain's average trip distance, which is 7 miles in 2010 according to National Travel Survey report (Anon, 2011a). This also tells that with regard to Scotland, average household income effect accounts more for the reduction in the equity VoT than average trip distance effect.

### 4.1.3 Results

Demand weighted average distance is used for all the zones and household incomes are still assigned the numbers generated for 707 zones. New local user benefit can be acquired. Similarly, employing demand weighted or total distance weighted average income and variable distances will output a new user benefit. Thus, user benefits for Scheme 1 can be illustrated in the table below.

**Table 8 Comparison of user benefits**

| Household income | Distance | User Benefit (£) |
|---|---|---|
| Scotland Specific Equity | Scotland demand weighted Equity | 843.42 |
| Scotland Specific Equity | Variable | 1297.14 |
| Variable | Scotland demand weighted Equity | 841.11 |
| Variable | Variable | 1298.07 |
| UK Equity | UK Equity | 1045.45 |
| Green book weights multiply corresponding benefits using local VoT | | 1115.63 |

It is of great interest when looking into these results. Using Scotland specific average income and variable distances outputs almost the same user benefit as implementing variable incomes and distances. Furthermore, the combination of Scotland specific average income and Scotland demand weighted average distance produces nearly the same benefit as varying incomes and fixing distance. This implies distance effects on VTTS are of great importance while income variations are less important in a Scottish context by looking at the first 4 rows. However, carrying out UK equity VoT brings a benefit that is 19.46% lower than actual local VoTs. In addition, it is 23.95% higher than

Scotland specific equity value of time. This confirms that Scotland is slightly poor in the UK context even though its average trip length is longer than UK, which results in the smaller equity value of time of Scotland.

### 4.1.4 Analysis of Scheme 2

Demand weighted and total distance weighted income are £45191.94 per year and £46760.16 per year separately. As a result, given demand weighted average distance is 10.26 miles, Scotland specific equity value of time is 7.89 pence/minute as these two weighted incomes belong to the same income band. Like Scheme 1, situations where fixed income & variable distance and variable incomes & fixed distance are examined, which yield user benefits listed in the table below.

**Table 9 User benefit comparison of Scheme 2**

| Household income | Distance | User Benefit (£) |
|---|---|---|
| Scotland Specific Equity | Scotland demand weighted Equity | -824.21 |
| Scotland Specific Equity | Variable | 330.79 |
| Variable | Scotland demand weighted Equity | -762.61 |
| Variable | Variable | 396.03 |
| UK Equity | UK Equity | -1021.64 |
| Green book weights multiply corresponding benefits using local VoT | | -272.34 |

Table above summarizes that user benefit estimated by combining Scotland specific average income and variable distances is lower (≈ 16.47%) than regional VoTs. On the contrary, if distances are fixed as whether Scotland demand weighted average or UK equity, this scheme will ruin the investment. Wasted money evaluated through incorporating Scotland specific average income and Scotland demand weighted average distance is slightly more (≈ 8.08%) than evaluation by means of variable incomes and Scotland demand weighted average distance but less (≈ 19.32%) than prevalent UK value of time. Again, varying or fixing incomes will not make a very big difference in a Scottish context. Instead, varying distances can cause the change in sign of benefit compared with fixing distances.

It's provoking to see this result as the benefit changes not only in size, but also in sign. This finding confirms that UK equity value of time may distort the investment. The reason why positive user benefit comes into being is that most time savings go to richer zones travelling longer distances that have higher VoTs, which results in the benefit Scheme 2 generates is bigger in absolute value than losses it causes.

## 4.2 Deriving Distributional Weights

Details about the population in each zone are also provided by Transport Scotland. According to Census 2011 (Anon, 2013a), average household size in Scotland is 2.19, hence, number of households in each zone can be estimated.

Distributional benefit equation can be expressed by: Equity benefit = weight (low income) * local benefit (low income) + weight (high income) * local benefit (high income) and a restriction is assumed in line with economic theory in order to determine a ratio of weight (low income) to weight (high income), which is weight (low income) > weight (high income).

As a result, test allocating 1 to low income or high income group until a ratio of weight (low income) to weight (high income) is bigger than 1.

Regarding Scheme 1, the median Scotland income of £35,356.78 per year is from zone ranked 342. Therefore, the advantaged group is zone ranked from 1 to 342, and the remainder is the

disadvantaged group. £26,135.25 per year and £50,203.74 per year are median household incomes for the advantaged and disadvantaged. As a consequence, the weights assigned to the advantaged and disadvantaged are 0.70 and 1.35, which implies a weight of 1.92 for the disadvantaged when setting the weight of the advantaged to 1. Distributional user benefit for the disadvantaged and the advantaged are £347.13 and £942.68, which outputs a weight of 0.30 for the disadvantaged if the advantaged is allocated a weight of 1 by applying the benefit of UK equity VoT. Hence, this again clarifies UK equity VoT brings about a lower weight for the disadvantaged. Subsequently, ratios of $W_L$ to $W_H$ by adopting 3 other benefits are displayed in the following table.

**Table 10 Comparison of ratios of WL to WH of Scheme 1**

| | Green Book | Scotland equity VoT | Scotland average income&variable distance | Variable income&Scotland average distance |
|---|---|---|---|---|
| Ratio of weight of the disadvantaged to weight of the advantaged | 1.92 | 1.90 | 0.99 | 1.92 |

Scotland equity VoT and variable income & Scotland average distance can produce almost the same ratio as the Green Book while Scotland average income & variable distance provides a lower ratio.

With respect to Scheme 2, the disadvantaged group suffers a loss of £782.83 while the advantaged group enjoys a benefit of £1,152.40, which attaches a weight of 2.78 to the disadvantaged provided the advantaged has a weight of 1 when employing UK equity VoT. This indicates UK equity VoT would result in a higher weight for the disadvantaged. And ratios by adopting other 3 benefits are illustrated in the table below.

**Table 11 Comparison of ratios of WL to WH of Scheme 2**

| | Green Book | Scotland equity VoT | Scotland average income&variable distance | Variable income&Scotland average distance |
|---|---|---|---|---|
| Ratio of weight of the disadvantaged to weight of the advantaged | 1.92 | 2.52 | 1.05 | 2.47 |

Scotland equity value of time and Variable income & Scotland average distance would lead to higher ratios while Scotland average income & variable distance outputs a lower ratio of $W_L$ to $W_H$ compared to the Green Book approach.

## 5 Conclusions

A lot of findings can be summarised and the most remarkable conclusions, which are helpful for advising decision making, are the following:

1) Employing the UK equity value of time will bring about much higher time saving benefits than regional values of time when trip lengths are short (<25 miles). This is even the case for London – although we find London enjoys the larges benefit compared to other regions and UK. Correspondingly benefits from inter-urban projects where trip distances are long are underestimated.
2) We find distance effects on the level of VTTS benefits are very important whereas income variations are less important. This is emphasised by our case studies where user benefits of one of the case studies increased by 24% using local VTTS, whilst the benefits of the second scheme went from negative to positive – primarily due to distance effects.
3) We find that the implied distributional weights associated with the equity value of time vary between applications, which leads to an inconsistency in the implied distributional weights associated with the equity VTTS.

These three principal findings undermine the argument that standard or equity values of time are fair – as clearly they are leading to biases between schemes that cater for long and short distance traffic which will lead to a spatial bias (urban vs inter-urban).

Clearly these findings are dependent to an extent on the method used to attribute household incomes to trips. Further research identifying how sensitive these findings are to the methods adopted here is needed. This would include both the examination of the data sources used and the statistical fitting of distributions (such as mixed Gaussian distribution). To a certain extent we were restricted due to the anonymity that was placed on the case studies, and as a consequence we were not able to use GIS data – of which some datasets may give better indicators regarding average incomes in a neighbourhood. Whilst we have raised issues regarding the inconsistency of the implied distributional weights in the equity VTTS we note that a move to a full distributional analysis would be very challenging in terms of the treatment of business user benefits and externalities such as safety and carbon. Notwithstanding these challenges there remains a need to better understand whether our current approach to distributional analysis is introducing unintended biases into the process – which this analysis appears to suggest – particularly in terms of the treatment of schemes which impact differently on short and long distance trips. Broadly speaking it appears that currently we are overestimating urban benefits and under-estimating inter-urban benefits.

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**Appendix A: Income distribution fitting**

**Table A.1 Fitting comparison for East Midlands, Yorkshire & Humberside**

| East Midlands | | Yorkshire & Humberside | |
|---|---|---|---|
| Distribution | Log likelihood | Distribution | Log likelihood |
| Birnbaum-Saunders | -1123.79 | Birnbaum-Saunders | -1140.19 |
| Exponential | -1138.57 | Exponential | -1151.36 |
| Extreme value | -1164.16 | Extreme value | -1176.62 |
| Gamma | -1124.96 | Gamma | -1138.67 |
| Generalized Extreme Value | -1125.29 | Generalized Extreme Value | -1139.73 |
| Inverse Gaussian | -1124.6 | Inverse Gaussian | -1141.64 |
| Logistic | -1145.58 | Logistic | -1158.86 |
| Log-Logistic | -1126.07 | Log-Logistic | -1140.37 |
| Lognormal | -1124.01 | Lognormal | -1139.34 |
| Nakagami | -1128.75 | Nakagami | -1141.78 |
| Negative Binomial | -1124.96 | Negative Binomial | -1138.67 |
| Normal | -1145.97 | Normal | -1158.89 |
| Poisson | -Inf | Poisson | -Inf |
| Rayleigh | -1135.06 | Rayleigh | -1148.8 |
| Rician | -1136.53 | Rician | -1150.85 |
| t location-scale | -1145.41 | t location-scale | -1158.93 |
| Weibull | -1126.9 | Weibull | -1140.13 |

**Table A.2 Fitting comparison for London and West Midlands**

| London | | West Midlands | |
|---|---|---|---|
| Distribution | Log likelihood | Distribution | Log likelihood |
| Birnbaum-Saunders | -1125.76 | Birnbaum-Saunders | -1129.75 |
| Exponential | -1135.92 | Exponential | -1140.56 |
| Extreme value | -1146.41 | Extreme value | -1164.98 |
| Gamma | -1124.07 | Gamma | -1128.04 |
| Generalized Extreme Value | -1128.28 | Generalized Extreme Value | -1129.29 |
| Inverse Gaussian | -1127.58 | Inverse Gaussian | -1131.25 |
| Logistic | -1141.84 | Logistic | -1147.85 |
| Log-Logistic | -1129.21 | Log-Logistic | -1129.94 |
| Lognormal | -1126.33 | Lognormal | -1128.91 |
| Nakagami | -1124.24 | Nakagami | -1130.95 |
| Negative Binomial | -1124.07 | Negative Binomial | -1128.04 |
| Normal | -1137.31 | Normal | -1147.69 |
| Poisson | -Inf | Poisson | -Inf |
| Rayleigh | -1130.09 | Rayleigh | -1137.86 |
| Rician | -1147.23 | Rician | -1140.11 |
| t location-scale | -1137.31 | t location-scale | -1147.69 |
| Weibull | -1124.11 | Weibull | -1129.4 |

**Table A.3 Fitting comparison for South West and Scotland**

| South West | | Scotland | |
|---|---|---|---|
| Distribution | Log likelihood | Distribution | Log likelihood |
| Birnbaum-Saunders | -1135.9 | Birnbaum-Saunders | -1131.33 |
| Exponential | -1145.34 | Exponential | -1141.74 |
| Extreme value | -1165.59 | Extreme value | -1166.22 |
| Gamma | -1132.96 | Gamma | -1129.9 |
| Generalized Extreme Value | -1135.34 | Generalized Extreme Value | -1131.37 |
| Inverse Gaussian | -1137.8 | Inverse Gaussian | -1132.86 |
| Logistic | -1151.39 | Logistic | -1150.29 |
| Log-Logistic | -1136.17 | Log-Logistic | -1132.03 |
| Lognormal | -1135.07 | Lognormal | -1130.7 |
| Nakagami | -1134.77 | Nakagami | -1132.66 |
| Negative Binomial | -1132.96 | Negative Binomial | -1129.91 |
| Normal | -1150.16 | Normal | -1149.64 |
| Poisson | -Inf | Poisson | -Inf |
| Rayleigh | -1141.25 | Rayleigh | -1140.21 |
| Rician | -1145.73 | Rician | -1143.15 |
| t location-scale | -1150.16 | t location-scale | -1149.64 |
| Weibull | -1133.75 | Weibull | -1131.17 |

**Table A.4 Fitting comparison for UK**

| UK | |
|---|---|
| Distribution | Log likelihood |
| Birnbaum-Saunders | -1125.55 |
| Exponential | -1134.89 |
| Extreme value | -1155.4 |
| Gamma | -1123.09 |
| Generalized Extreme Value | -1125.32 |
| Inverse Gaussian | -1127.35 |
| Logistic | -1142.35 |
| Log-Logistic | -1126.08 |
| Lognormal | -1124.79 |
| Nakagami | -1125.01 |
| Negative Binomial | -1123.09 |
| Normal | -1140.69 |
| Poisson | -Inf |
| Rayleigh | -1131.95 |
| Rician | -1137.31 |
| t location-scale | -1140.69 |
| Weibull | -1123.95 |

In addition, figures below illustrate the level of fitting, where five best distributions are plotted.

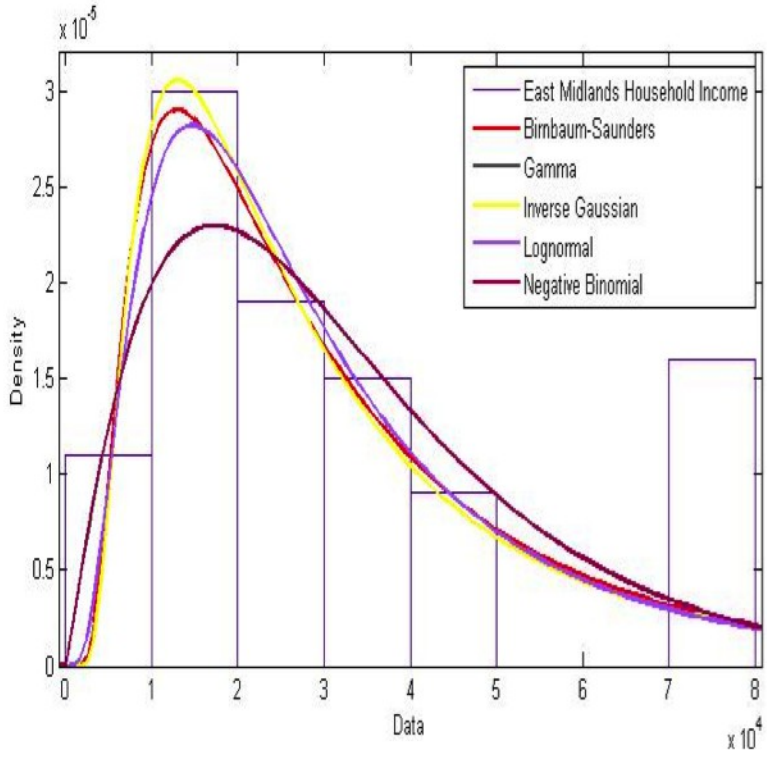


**Figure A.1 Distribution fitting for East Midlands**

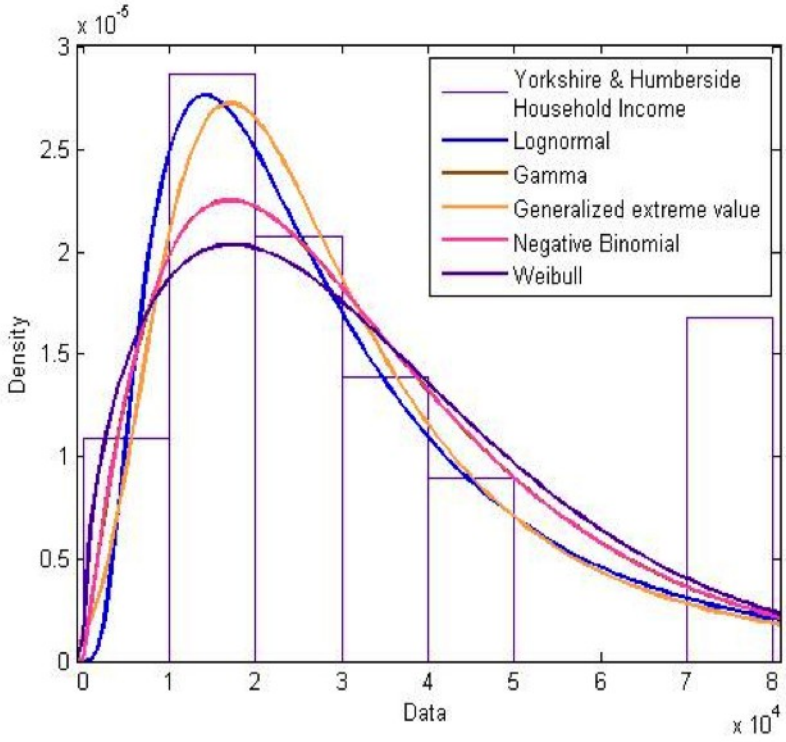


**Figure A.2 Distribution fitting for Yorkshire & Humberside**

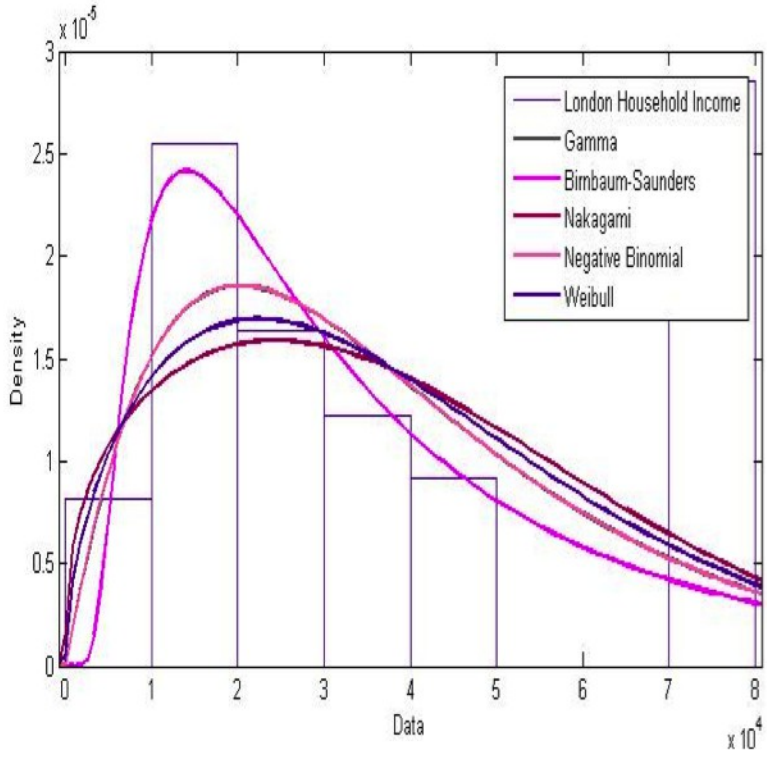


**Figure A.3 Distribution fitting for London**

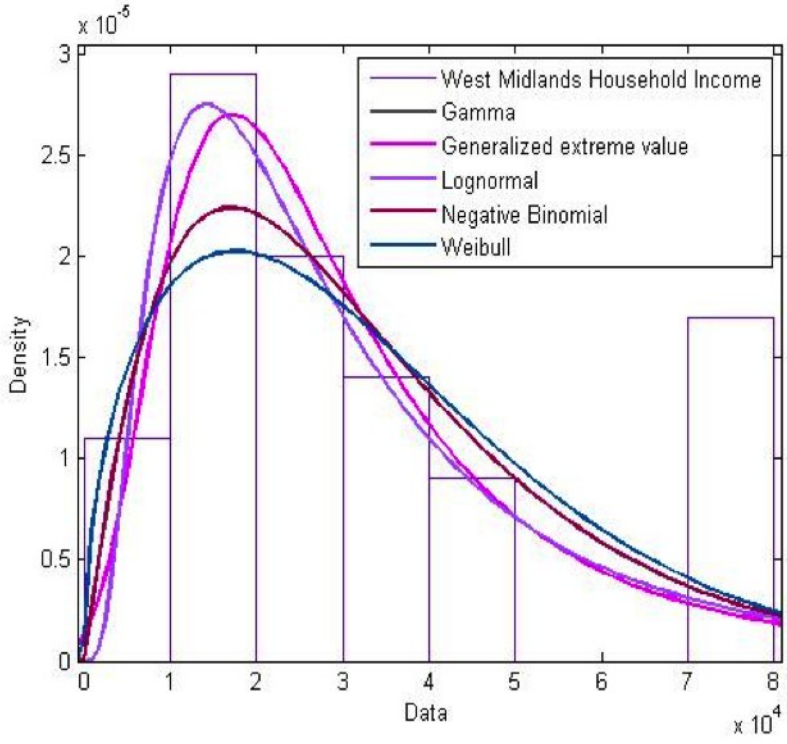


**Figure A.4 Distribution fitting for West Midlands**

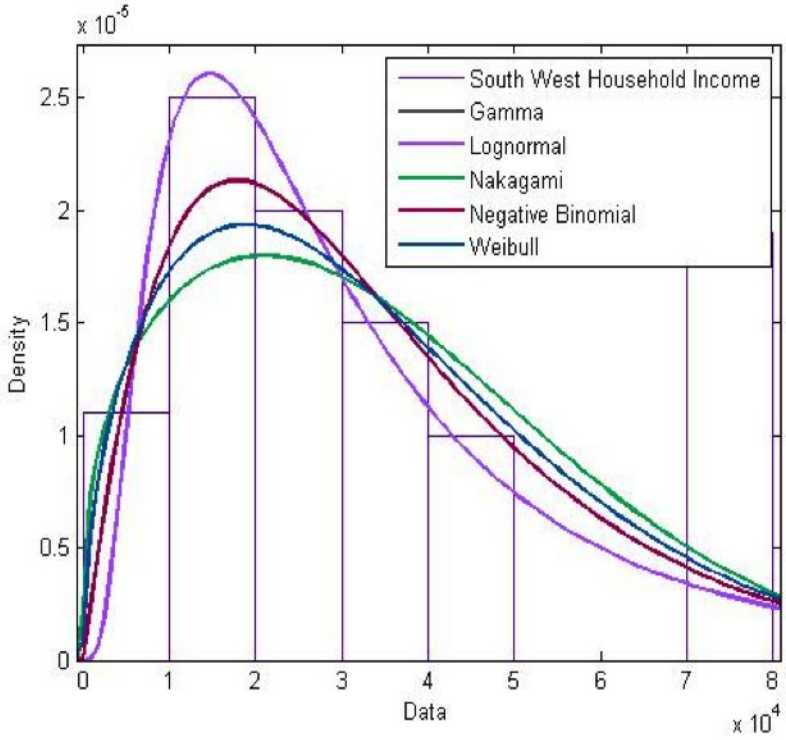


**Figure A.5 Distribution fitting for South West**

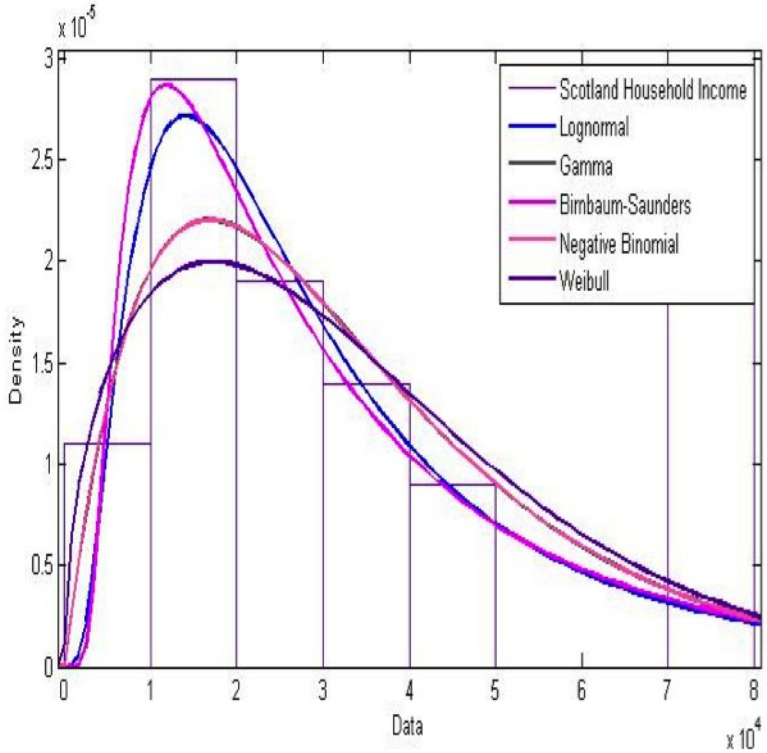


**Figure A.6 Distribution fitting for Scotland**

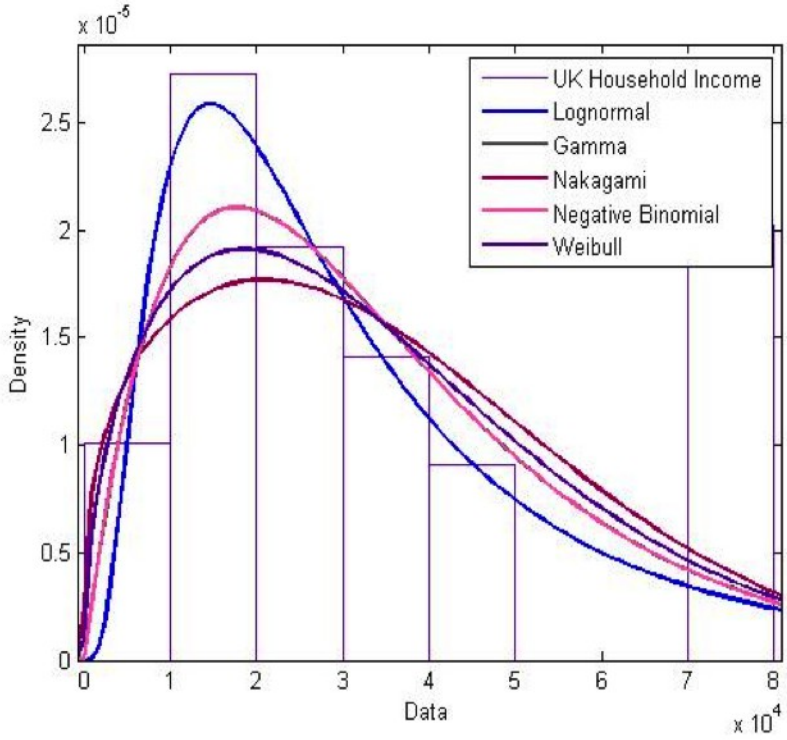


**Figure A.7 Distribution fitting for UK**

Even though Negative Binomial distribution has the same log likelihood as Gamma distribution for Yorkshire and Humberside, London, West Midlands and South West and UK, the latter is implemented for generating random incomes as Negative Binomial is for discrete data while Gamma is for continuous data. Apart from Birnbaum-Saunders distribution fitting household incomes of East Midlands best, the rest are best fitted by Gamma distribution, which agrees with Wootton and Pick's opinion that *a Gamma distribution fits the observed data well and is analytically convenient* (Wootton and Pick, 1967).

Key parameters for Birnbaum-Saunders and Gamma distributions generated by MATLAB are given in tables below.

**Table A.5 Key parameters for Birnbaum-Saunders distribution of East Midlands**

| | Mean | Variance | Beta | Gamma |
|---|---|---|---|---|
| Value | 32321.70 | 6.54E+08 | 24820.70 | 0.78 |

**Table A.6 Key parameters for Gamma distribution**

| | Mean | Variance | Shape parameter | Scale parameter |
|---|---|---|---|---|
| Yorkshire and Humberside | 32846.50 | 5.22E+08 | 2.07 | 15887.10 |
| London | 39775.50 | 7.78E+08 | 2.03 | 19550.70 |
| West Midlands | 33045.00 | 5.29E+08 | 2.06 | 16008.70 |
| South West | 34663.00 | 5.85E+08 | 2.05 | 16870.80 |
| Scotland | 33438.00 | 5.54E+08 | 2.02 | 16577.60 |
| UK | 35016.20 | 6.06E+08 | 2.02 | 17312.70 |

**Appendix B – Fitting distribution for case studies**

The following is the table showing the comparison of 17 distributions used to fit.

**Table B.1 Comparison of distribution fitting**

| Distribution | Log likelihood |
|---|---|
| Birnbaum-Saunders | -8009.54 |
| Exponential | -8274.83 |
| Extreme value | -8384.57 |
| Gamma | -8050.78 |
| Generalized Extreme Value | -7997.39 |
| Inverse Gaussian | -8006.42 |
| Logistic | -8165.75 |
| Log-Logistic | -8025.56 |
| Lognormal | -8012.63 |
| Nakagami | -8103 |
| Negative Binomial | -8050.79 |
| Normal | -8204.73 |
| Poisson | -Inf |
| Rayleigh | -8103.22 |
| Rician | -8325.28 |
| t location-scale | -8137.31 |
| Weibull | -8097.46 |

Obviously, the Generalized Extreme Value distribution is adopted to generate 707 random household incomes for each zone, which can be displayed in the figure below.

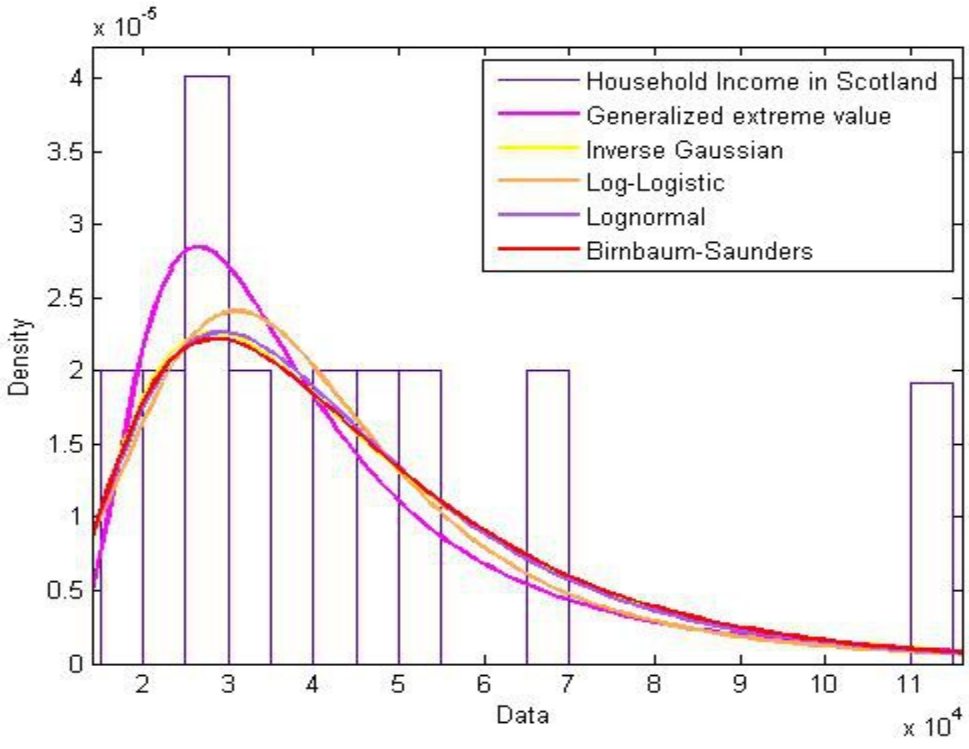

**Figure B.1 Distribution fitting of Scottish household income**

And its key parameters are illustrated in the table below.

**Table B.2 Key parameters of GEV distribution**

| | Mean | Variance | k | sigma | mu |
|---|---|---|---|---|---|
| Value | 46097.9 | 1.93E+09 | 0.366916 | 13718 | 30472.6 |

However, a glance at the 707 household incomes for each zone generated finds that its minimum income is £10,886.16 per year and maximum one is £479,217.14 per year with an average income of £44,664.73 per year. The reason why mean household income is not very close to £46,097.9 per year is because the number of simulations generated is not enough to prevent random fluctuations in the data.